تمرین های کاربرد انتگرال مختلط در محاسبه انتگرال های معین

توابع گویای مثلثاتی

$$\int_0^{2\,\pi} \frac{\cos(x)^2}{13+12\cos(x)}\,dx = \frac{13}{45}\,\pi$$

$$\int_0^{2\,\pi} \frac{\cos(x)^4}{1+\sin(x)^2}\,dx = \frac{\pi\,(4\,\sqrt{2}-5)}{(\sqrt{2}+1)\,(\sqrt{2}-1)}$$

$$\int_0^{2\,\pi} \frac{\sin(x)^2}{\frac{5}{4}-\cos(x)}\,dx = \pi$$

$$\int_0^{2\,\pi} \frac{1}{\sin(x)^2+4\cos(x)^2}\,dx = \pi$$

$$\int_0^{2\,\pi} \frac{1}{13+12\sin(x)}\,dx = \frac{2}{5}\,\pi$$

$$\int_0^{2\,\pi} \frac{1}{2+\cos(x)}\,dx = \frac{2}{3}\,\pi\,\sqrt{3}$$

$$\int_0^{2\,\pi} \frac{1}{(2+\cos(x))^2}\,dx = \frac{4}{9}\,\pi\,\sqrt{3}$$

توابع گویای ناسره

$$\int_{-\infty}^{\infty} \frac{x^2+1}{x^4+1}\,dx = \sqrt{2}\,\pi$$

$$\int_{-\infty}^{\infty} \frac{x^2-1}{(x^2+1)^2}\,dx = 0$$

$$\int_{-\infty}^{\infty} \frac{x^2-x+2}{x^4+10\,x^2+9}\,dx = \frac{5}{12}\,\pi$$

$$\int_{-\infty}^{\infty} \frac{1}{(x^2+1)\,(x^2+4)^2}\,dx = \frac{5}{144}\,\pi$$

$$\int_{-\infty}^{\infty} \frac{x}{(x^2+4\,x+13)^2}\,dx = -\frac{1}{27}\,\pi$$

انتگرال های ناسره فوریه

$$\int_{-\infty}^{\infty} \frac{e^{(I\,x)}}{x\,(x^2+1)}\,dx = 2\,I\,\pi\,(\frac{1}{2} - \frac{1}{2}\,e^{(-1)})$$

$$\int_{-\infty}^{\infty} \frac{e^{(I\,x)}}{x^2+1}\,dx = \pi\,e^{(-1)}$$

$$\int_{-\infty}^{\infty} \frac{e^{(I\,x)}}{x^2+4\,x+20}\,dx = \frac{1}{4}\,\frac{\pi\,e^{(-4)}}{(e^I)^2}$$

$$\int_{-\infty}^{\infty} \frac{e^{(I\,x)}}{x^2-5\,x+6}\,dx = 2\,I\,\pi\,(-\frac{1}{2}\,(e^I)^2 + \frac{1}{2}\,(e^I)^3)$$

$$\int_{-\infty}^{\infty} \frac{e^{(I\,x)}}{x}\,dx = I\,\pi$$

$$\int_{-\infty}^{\infty} \frac{x\,e^{(I\,x)}}{x^2+4\,x+20}\,dx = \frac{1}{4}\,\frac{\pi\,(4\,I\,e^{(-4)} - 2\,e^{(-4)})}{(e^I)^2}$$

$$\int_{-\infty}^{\infty} \frac{x\,e^{(I\,x)}}{x^2+9}\,dx = I\,\pi\,e^{(-3)}$$